

# Millimeter wave measurements of temperature dependence of complex permittivity of GaAs plates by a circular waveguide method

Takashi Shimizu and Yoshio Kobayashi

Dept. of Electrical and Electronic Systems, Saitama University, Urawa, Saitama 338-8570, Japan

**Abstract** —A circular waveguide method is improved to measure the complex permittivity of low loss dielectric plates using the  $TE_{0ml}$  mode with integer  $m$  of the higher order accurately in the millimeter wave range. The measurement principle is described on the basis of a rigorous analysis by the mode matching technique. A mode chart presented is effective to identify many resonance modes observed in the measurement. The temperature dependences for GaAs plates were measured at 26GHz for the  $TE_{011}$  mode and at 40GHz for the  $TE_{021}$  mode. It is verified that this method is useful to measure the temperature dependence precisely. Moreover it is expected to be able to evaluate the quantity of lattice defects of GaAs crystal.

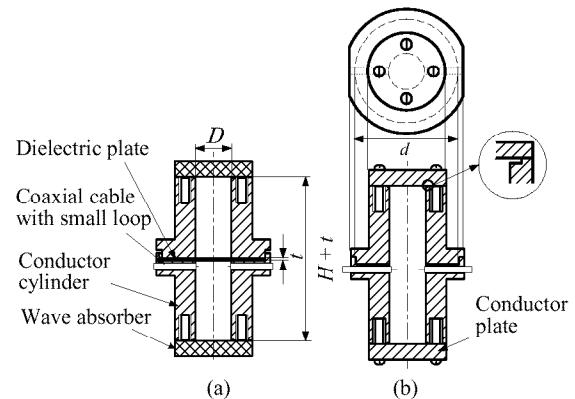
## I. INTRODUCTION

Open resonator methods are known as an effective technique for measuring low loss dielectric materials accurately in the millimeter wave range 30-300GHz [1][2]. However these methods are not suitable to measure the temperature dependences, because a measurement apparatus is mechanically unstable for temperature change in a cryostat or an oven.

On the other hand, a  $TE_{011}$  mode circular waveguide method proposed by S.B.Cohn and K.C.Kelly [3], where a resonator is constituted by inserting a circular disk sample into a  $TE_{01}$  mode cutoff circular waveguide, was applied to millimeter wave measurement [4]. Further, the resonator structure by this method was improved to measure any size of a plate sample nondestructively [5][6]. However the measurement of loss tangent  $\tan\delta$  has not been discussed sufficiently [5]. Further, It is not practical to adjust the coupling strength by a waveguide [6].

Currently, we can use millimeter wave vector network analyzers constituted by a coaxial cable system. The coaxial cable is expected to be easy to adjust the coupling strength finely.

In this paper, the circular waveguide method is improved to measure the temperature dependences of complex permittivity of low loss dielectric plates at 50GHz. The  $TE_{0ml}$  mode with integer  $m$  of the higher order is used to make the resonance frequency close to a resonance frequency where the effective relative conductivity  $\sigma_r$  is determined from a measured unloaded  $Q$ ,  $Q_u$  for a circular empty cavity, because the resonance



$$\det H(f_0; \epsilon_r, \epsilon_g, t, g, D, H, d) = 0 \quad \dots \quad (1)$$

$$\tan\delta = A/Q_u - BR_s \quad \dots \quad (2)$$

where

$$A = -f_0 / [2\epsilon_r \cdot (\Delta f_0 / \Delta \epsilon_r)] \quad \dots \quad (3)$$

$$B = \frac{1}{120\pi k_0 \epsilon_r} \frac{1}{(\Delta f_0 / \Delta \epsilon_r)} \left( \frac{\Delta f_0}{\Delta H} + \frac{\Delta f_0}{\Delta R} + \frac{\Delta f_0}{\Delta g} \right) \dots \quad (4)$$

$$R_s = \sqrt{\pi f_0 \mu_0 / \sigma} \quad (\Omega) \quad \mu_0 = 4\pi \times 10^{-7} \quad (H/m)$$

$$\sigma = \sigma_0 \sigma_r \quad \sigma_0 = 58 \times 10^6 \quad (S/m)$$

(c)

Fig. 1. Cross sectional view of a resonator structure.

(a) Circular cylinder clamping a dielectric plate.

(b) Circular empty cavity.

(c) Measurement formulas.

frequency of the  $TE_{011}$  mode decreases considerably when a sample plate is thick or with high permittivity. An automatic measurement system using coaxial cables is also developed to measure the temperature dependence efficiently. By this method, the temperature dependences of complex permittivity for GaAs plates will be measured at 26GHz for the  $TE_{011}$  mode and at 40GHz for the  $TE_{021}$  mode.

## II. MEASUREMENT PRINCIPLE

A resonator structure used in this measurement is shown

in Fig. 1(a). The circular cylinder is cut into two parts in the middle of the height  $H$ . A dielectric plate sample of the thickness  $t$ , which is a larger size than the diameter  $D$ , is placed between these cylinders and clamped by two clips; hence a sample to be measured can be quickly removed and replaced by another one. The cylinders constitute  $TE_{0m}$  mode cutoff waveguides; hence the fields decay exponentially on either side of the sample. Wave absorbers are needed to eliminate the unnecessary cavity modes.

The values of  $D$ ,  $H$  and  $\sigma_r$  of a copper cavity are measured by a circular empty cavity shown in Fig. 1(b). In this case, copper plates are attached at both ends in place of the wave absorbers. The degenerate  $TM_{11p}$  mode can be separated from the  $TE_{01p}$  mode by grooves machined at both ends of the cylinders. The  $D$  and  $H$  value were calculated from two resonance frequencies for the  $TE_{01p}$  and  $TE_{01q}$  ( $p \neq q$ , integer) modes and the effective relative conductivity  $\sigma_r$ , including influence of oxidation and roughness of the copper surface, was determined from a measured unloaded  $Q$ ,  $Q_u$  for the  $TE_{01p}$  mode [7].

These resonators are excited and detected at the middle of the cylinder by a pair of UT-47 semi-rigid coaxial cables (outer diameter 1.2mm) with small loops at the top.

The axially symmetric  $TE_{0ml}$  ( $m=1, 2, \dots$ ) modes, as shown in Fig. 2, are used to avoid air gap effects at the plate-cylinder interface. The value of relative permittivity

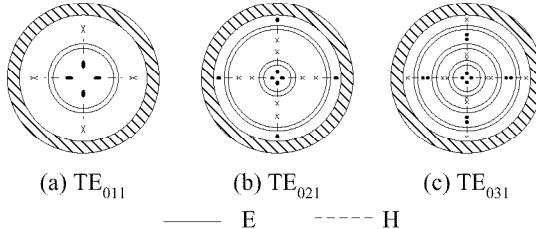


Fig. 2. Fields plots of  $TE_{011}$ ,  $TE_{021}$  and  $TE_{031}$  modes.

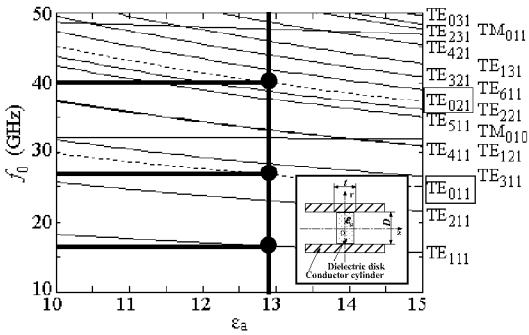


Fig. 3. Mode chart of a dielectric disk resonator loaded in a circular waveguide calculated for  $D=6.991\text{mm}$  and  $t=0.607\text{mm}$  (GaAs plate).

$\epsilon_r$  and loss tangent  $\tan\delta$  of the sample, in consideration of the fringe effect, can be calculated accurately from the measured value of  $f_0$  and  $Q_u$  of the  $TE_{0m1}$  mode, by using measurement formulas based on rigorous analysis by the mode matching technique, as shown in Fig. 1(c), where constants  $A$  and  $B$  are calculated from the frequency changes due to each perturbation by using eq. (1) [8].

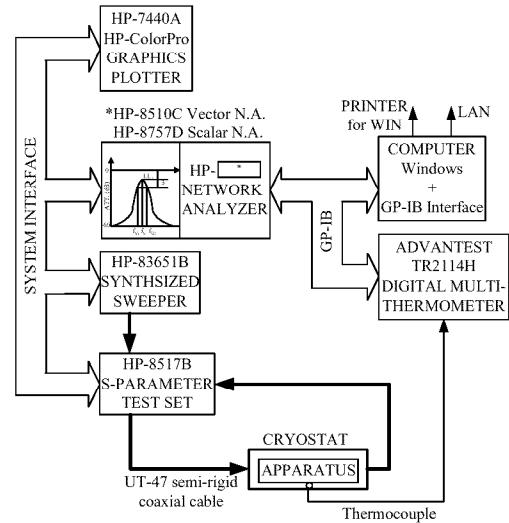


Fig. 4. Automatic measurement system.

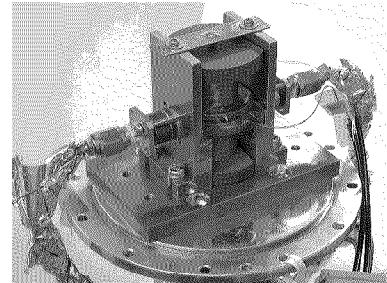


Fig. 5. Photograph of measurement apparatus attached on a cryostat.

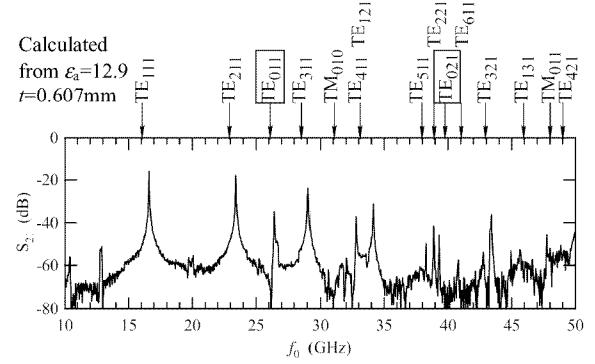
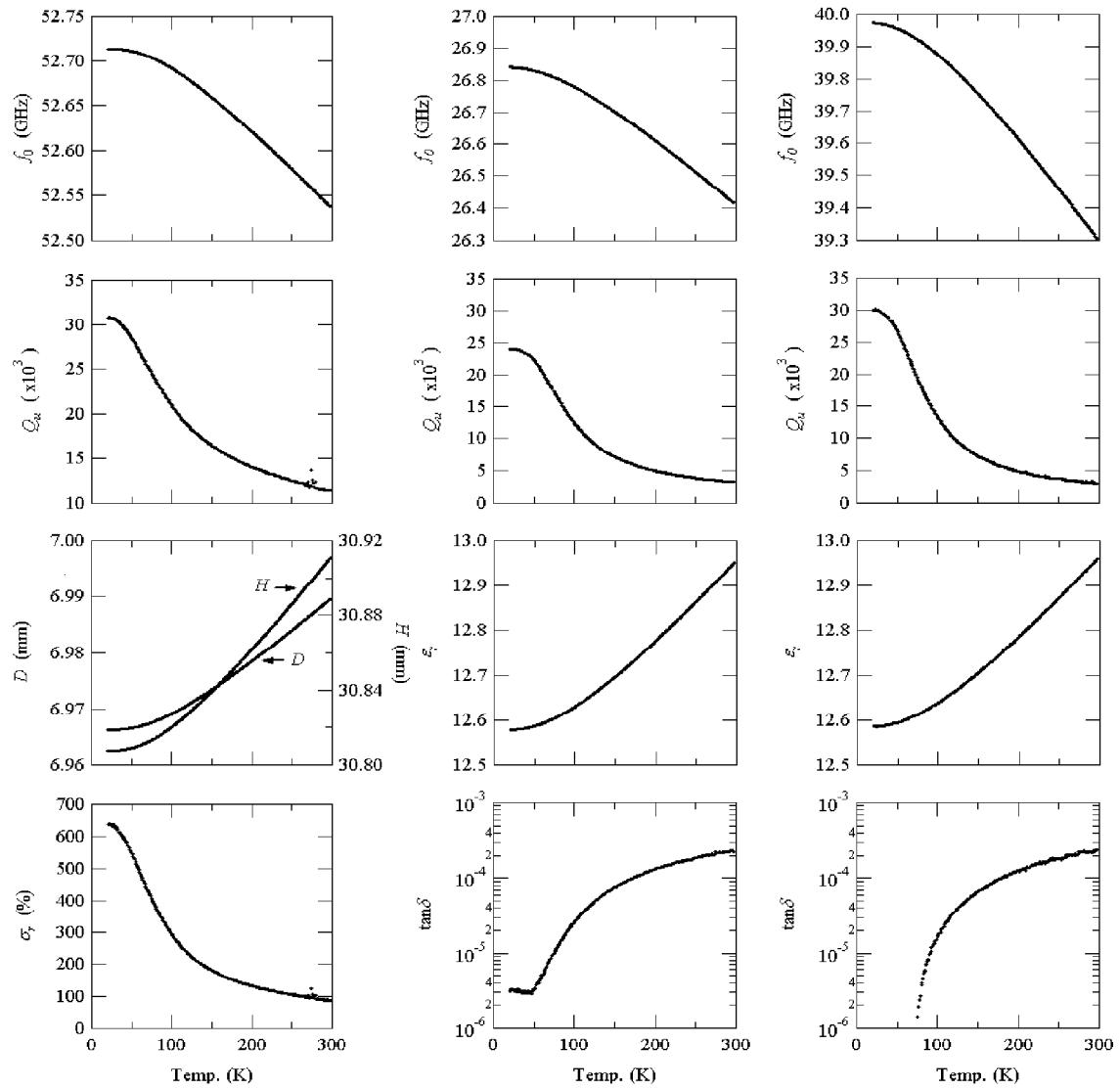


Fig. 6. Frequency response measured for a GaAs plate and resonance frequency calculated from mode chart.

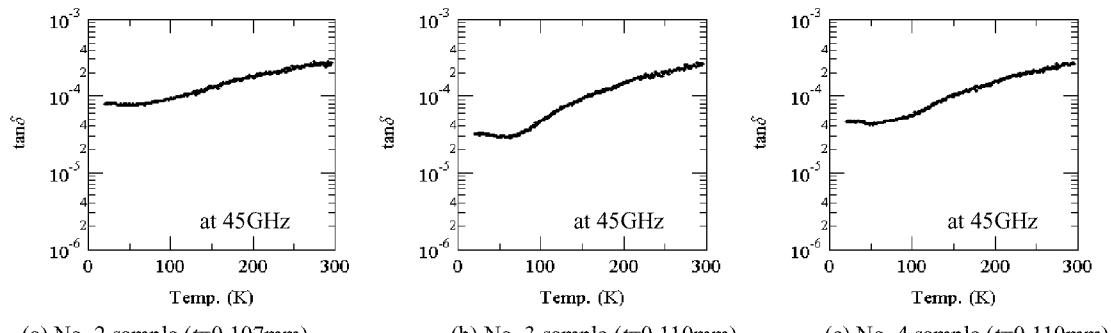


(a) Circular empty cavity for TE<sub>011</sub>

(b) GaAs plate for TE<sub>011</sub>

(c) GaAs plate for TE<sub>021</sub>

Fig. 7. Temperature dependences of circular empty cavity and GaAs plate with  $t=0.607\text{mm}$  (No. 1 sample).



(a) No. 2 sample ( $t=0.107\text{mm}$ )

(b) No. 3 sample ( $t=0.110\text{mm}$ )

(c) No. 4 sample ( $t=0.110\text{mm}$ )

Fig. 8. Temperature dependences of  $\tan\delta$  for some GaAs plates measured using the TE<sub>011</sub> mode.

A program making a mode chart for  $f_0$  versus  $\epsilon_a$  was developed, where  $f_0$  is the measured resonance frequency and  $\epsilon_a$  is an approximate relative permittivity when neglecting the fringe effect. The mode chart for  $D=6.991$  mm and  $t=0.607$  mm for a GaAs plate sample is shown in Fig. 3. This is more powerful to identify resonance modes. In this case, it is found quickly that  $\epsilon_a$  is 12.9 from  $f_0$  of the dominant  $TE_{111}$  mode and resonance frequencies of the  $TE_{011}$  and  $TE_{021}$  modes appear around 26GHz and 40GHz, respectively.

### III. AUTOMATIC MEASUREMENT SYSTEM FOR TEMPERATURE DEPENDENCE

An automatic measurement system was developed to measure the temperature dependence more efficiently in millimeter wave region. It was constructed by using 2.2mm semi-rigid coaxial cables and V connectors. This block diagram is shown in Fig. 4.

A measurement apparatus machined from copper in Fig. 1(a) is set in a cryostat and a thermocouple is attached on the middle of the cylinder, as shown in Fig. 5. The cryo-cooler is turned off to measure  $f_0$  and  $Q_u$  without mechanical vibrations, after it is cooled down from the room temperature to 20K. Using programs for Windows personal computer developed in our laboratory,  $f_0$  and  $Q_u$  are measured automatically at each temperature shift of 1K. A program, which was  $\epsilon_r$  and  $\tan\delta$  calculated from measured  $f_0$  and  $Q_u$ , was developed on the basis of eqs. (1)-(4).

The frequency response for a GaAs plate measured by using this system is shown in Fig. 6. Mode identification of these measured resonance peaks can be performed from the resonance frequencies calculated from the mode chart, indicated on the top of Fig. 6.

### IV. MEASURED RESULTS

In advance to measure the temperature dependence of  $\epsilon_r$  and  $\tan\delta$  of plate samples, we need to measure the temperature dependence of  $D$ ,  $H$  and  $\sigma_r$  of the empty cavity. The measured results for  $TE_{011}$  mode are shown in Fig. 7(a).

The temperature dependence of  $\epsilon_r$  and  $\tan\delta$  of a GaAs plate ( $10 \times 10$  mm $^2$ ,  $t=0.607$  mm and coefficient of linear thermal expansion  $\tau=6.9$  ppm/K) were measured using the  $TE_{011}$  mode at 26GHz and using the  $TE_{021}$  mode at 40GHz. The measured results are shown in Figs. 7(b) and (c). In Fig. 7(c), The value of  $\tan\delta$  under 75K is smaller than  $10^{-6}$  below the measurement limit of  $\tan\delta$  in this method. The measured results of  $\tan\delta$  for the other three GaAs plates with  $t=0.107$ , 0.110 and 0.110mm are shown in Fig. 8. The values of  $\tan\delta$  in Fig. 8 are the same as in Fig. 7 at room temperature. However the  $\tan\delta$  values took the

different ones under 100K. It appears that this depends on the quantity of the lattice defects of GaAs crystal. Therefore, we can expect to be able to evaluate it from the behavior of  $\tan\delta$  under 100K

### V. CONCLUSION

It was verified that an improved circular waveguide method discussed in this paper is useful to measure the temperature dependence of complex permittivity accurately and efficiently for low loss dielectric plate samples in the millimeter wave range. The measurement precisions of this method are 0.8 percents for  $\epsilon_r$  and 5 percents for  $\tan\delta$ .

We can expect to apply this measurement method till 100GHz by using a cavity structure of  $D=3$  mm.

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